demand for their services and, by the same token, their profitability. The second term, which stems from the difference in installed bases, also increases for the small backbone but *decreases* for the big one with the quality of interconnection. This term illustrates the strategic impact of a degradation of interconnection quality. Although the demands for both backbones decrease when this quality goes down, the smaller backbone is more affected. Hence, by degrading the interconnection quality, the bigger backbone can "discourage" its rival from attracting new customers, thereby increasing its own residual demand.

One therefore has

$$\frac{\partial \pi_1^*}{\partial \theta} < \frac{\partial \pi_2^*}{\partial \theta},$$

the bigger backbone has less incentives than its rival to maintain a high quality of interconnection.

The strategic effect of degrading the quality of interconnection is illustrated by Figure 1a, which represents the two backbones' reaction curves (the horizontal axis corresponds to q_1 and the vertical axis to q_2, and the steeper curves are the dominant backbone's reaction curves, $q_1 = R_1(q_2)$) for particular values of the parameters, respectively for $\theta = 1$ (solid lines) and $\theta = 1/2$ (dashed lines). In the absence of strategic interaction (that is, keeping the rival's capacity constant), each backbone suffers from a degradation of the quality of interconnection: both reaction curves move "downwards" when θ decreases).[4] But this negative impact is asymmetric: it is smaller for the bigger backbone ($R_1(.)$ moves less than $R_2(.)$), so that the equilibrium point (the intersection between the two reaction curves) moves away from (and below) the 45° line; the dominant backbone's capacity is thus less affected than its rival's one (in this example, it is almost not affected when θ decreases from 1 to 1/2), which in turn implies that the bigger backbone has less incentives to maintain the quality of interconnection. It may even be the case that the dominant backbone *benefits* from the degradation of the interface: in this example, the dominant backbone ends up attracting more customers when $\theta = 0$ than when $\theta = 1$, whereas the smaller backbone almost disappears from

[4] "Being hurt" applies here to both market shares and profits. Profits satisfy $\pi_i = (1-v)\,q_i^2$ as long as backbone i remains on its reaction curve, so that the evolution of the capacities q_i indeed suffices to determine whether profits increase or decrease with the quality of interconnection (in other words, $q_i = R_i(q_j;\theta)$ summarizes both the direct impact of θ and the impact of a change in the rival's quantity).

the market for unattached customers when $\theta = 0$.

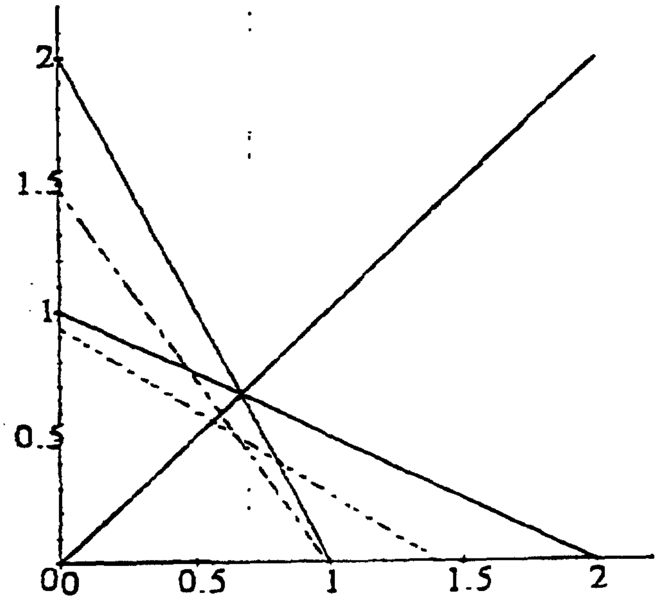

Figure 1a
Effect of a degradation from $\theta = 1$ (solid lines) to $\theta = 1/2$ (dashed lines)
$(c = 0, v = .4, \beta_1 = .4, \beta_2 = .1)$

The impact of a degradation of the interconnection is even more asymmetric when this interconnection is already degraded, since this increases the dominant backbone's advantage: the difference in "total bases", $\beta_1 - \beta_2 + q_1 - q_2$, increases when interconnectivity is degraded. There is thus a "snowball effect", making further degradations less harmful or more attractive for the dominant backbone; for instance, in the above example the dominant backbone, who is slightly hurt when θ decreases from 1 to 1/2, benefits from another reduction from 1/2 to 0.

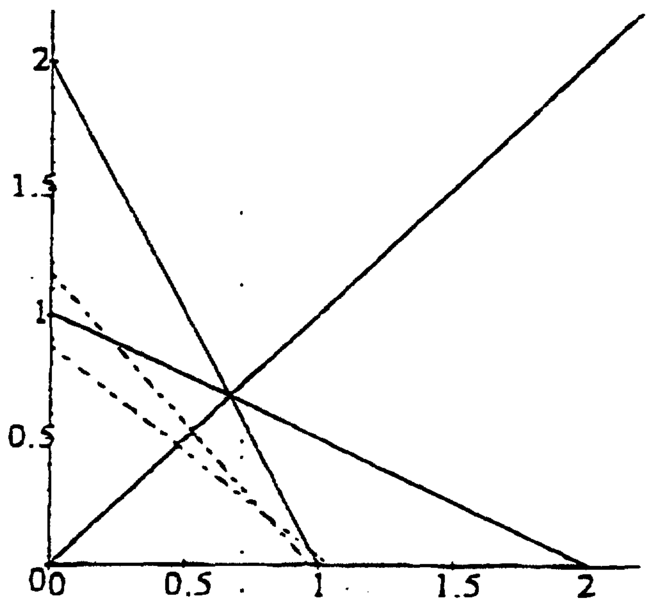

Figure 1b

Effect of a degradation from $\theta = 1$ (solid lines) to $\theta = 0$ (dashed lines)

$(c = 0, v = .4, \beta_1 = .4, \beta_2 = .1)$

In the absence of cost for maintaining the quality of interconnection, the smaller backbone always prefers to maintain a perfect interconnection. Similarly, the two backbones would have an incentive to maintain a high interconnection quality if they were of the same size. When one backbone has a bigger installed base, however, he may well prefer to degrade the interconnection quality. This is indeed the case when the strategic effect dominates (as in Figure 1), that is, when the installed base advantage is large enough. Furthermore, because of the snowball effect mentioned above, the dominant backbone's best degradation strategy, if degradation is indeed optimal for the dominant network, will be to refuse interconnection ($\theta = 0$).[5]

[5] Starting from perfect quality of interconnection ($\theta = 1$), $\frac{d\pi_1^*}{d\theta}$ is negative when

$$9\Delta > 3\beta + 2\frac{1 - c + v\beta}{1 - v},$$

In the presence of a cost of quality of interconnection, it is in general neither socially efficient nor privately optimal, even for the smaller backbone to choose a perfect connectivity ($\theta = 1$). But the larger backbone has a bias towards lower quality, that is, a dominant supplier always favors a lower quality of interconnection than the smaller backbone. Moreover, the dominant backbone's best strategy will still be to refuse interconnection ($\theta = 0$) if it is optimal to do so in the absence of a cost of quality of interconnection.

Remark (Cournot versus Bertrand competition). It is well-known that oligopoly models sometimes generate substantially different conclusions, depending on the assumptions made on the nature of competitors' strategic interactions. One particular example is the contrast between Cournot models, which take quantities or capacities as the key strategic variables, and Bertrand models, which instead consider prices to be the key strategic decisions: capacities are often considered to be strategic substitutes (one firm will react to its rival's expansion by *reducing* its own quantity or capacity) whereas prices are usually considered to be strategic complements (a firm will react to increases in its rivals' prices by *increasing* its own price). However, our analysis of the dominant firm's strategic incentive to degrade interconnectivity is robust to changes in the modelling of the day-to-day mode of competition: We have explored an alternative duopoly model where the two backbones compete in prices rather than in capacities, and shown that the same analysis as above exactly applies to this case as well: (i) it is still true that the larger backbone has less incentives to invest in the quality of interconnection; (ii) assuming away the costs of interconnection, a small backbone would always favor the best quality of interconnection, whereas the dominant backbone may well prefer instead a small degradation, and even more so, a complete degradation of this quality. This alternative model, which allows for some differentiation between the two networks, further shows that the analysis remains valid even when some of the new customers have an *a priori* bias in favor of the smaller backbone.

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that is, when the big backbone's advantage in installed base is large enough. However, the profit π_1^c is convex with respect to the interconnection quality θ. Hence, the bigger backbone's optimal choices would either be $\theta = 1$ (perfect interconnection) or $\theta = 0$ (no interconnection). The comparison of π_1^c and π_1^{nc}, the profits obtained for each of those two choices, shows that the bigger backbone will prefer not to interconnect ($\pi_1^{nc} > \pi_1^c$) when

$$\Delta > \frac{(1-2v)\,(2\,(1-c)+(3-v)\,\beta)}{3\,(1-v)\,(3-2v)}.$$

When for example $c = 0$ and $v = 1/3$, this condition boils down to $\Delta > 1/4$.

3.5. Multihoming

Until now, we have assumed that customers could only "single home". A new customer with type ν then obtains net surplus

$$\nu - \bar{p}.$$

Can it be in the interest of the customer to multihome, assuming this is feasible (see our discussion in the text)? The answer is negative: multihoming yields net utility

$$\begin{aligned} \nu + v(\beta_1 + q_1 + \beta_2 + q_2) - (p_1 + p_2) &= \nu - 2\bar{p} - v\theta(\beta_1 + q_1 + \beta_2 + q_2) \\ &< \nu - \bar{p}, \end{aligned}$$

since $\bar{p} \geq 0$ (that is, $q_1 + q_2$ cannot exceed the total demand, equal to 1); multihoming is dominated, because consumers would end-up paying twice.

Remark. It is certainly true that the fact that consumers' preferences for connectivity (*per unit of Internet usage*) are the same for all customers, creates little scope for multihoming. More heterogeneity with respect to the benefits of connectivity per unit of usage *might* generate some multihoming, but its extent is likely to be limited.

Remark. In our model, customers pay a fixed fee for Internet usage. Wouldn't usage based pricing be more favorable to multihoming? It is not necessarily so, for two reasons:

1. If the level of degradation is high, (θ is equal or close zero), usage-based pricing does not change the analysis. Intuitively, for high levels of degradation, usage is on-net usage, and so multihoming amounts to paying $p_1 + p_2$ even under usage-based pricing. In other words, multihoming leads to a total usage below the sum of the two usages under singlehoming only if there is a substantial off-net activity. Our emphasis on total degradation implies that multihoming is still suboptimal under usage-based pricing.

2. As we noted in the text, degradation and non-linear pricing are complementary strategies for the dominant network. The fixed fee is an extreme example of a price discount. More generally, the dominant network might accompany degradation with some amount of non-linear pricing.

4. Merger analysis

We now extend the Cournot model and consider a situation where initially four backbones are competing on an equal basis; that is, they each have an installed base of the same size,

$$\beta_i = \frac{\beta}{4}.$$

We first briefly consider this initial situation, and then analyze the outcome of a merger between two of them, which creates a dominant position: the new merged firm has half of the installed base, whereas the other two still only have one fourth each of the total installed base. For computational simplicity, we assume that each backbone may (unilaterally) choose to be compatible with each of the other backbones ($\theta_{ij} = 0$ or 1), and we keep our earlier assumption that compatibility requires cooperation of both parties. As we will later discuss, we will further allow the backbones to limit the traffic at the interfaces even when they are compatible ($\theta_{ij} = 1$).

We show that in the pre-merger situation, all backbones have an incentive to maintain the highest quality of interconnection. In the post-merger situation, the small backbones again have an incentive to maintain a high quality of interconnection, but the dominant backbone may not want to. To analyze this issue, we consider three possible strategies for the dominant backbone:[8]

- *Accommodation*: The dominant backbone does not degrade the quality of interconnection with any other backbone; the industry is governed by ubiquitous peering agreements.
- *Global degradation*: The dominant backbone decides not to interconnect at all. It is then optimal for the other backbones to enter into a peering agreement.
- *Targeted degradation*: The dominant backbone refuses to deal with one of the other two *and* further limits the interface capacity with the other backbone, so that the quality of this interface is excellent (we will assume "perfect") if this backbone does not offer transit to the other, but abysmal (we will assume "equal to 0") if the two small backbones enter a transit agreement.

Constraining the interface with the non-targeted backbone: an illustration. To illustrate the limitation of the interface with the non-targeted backbone, let us return to the interpretation of the interconnection quality parameter as the interface capacity. Assume for simplicity that all messages have the same value v (so $w \equiv v$). Let μ denote the capacity of the interface and t_{ij} denote the potential traffic between backbones i and j ($t_{ij} \equiv 2(\beta_j + q_i)(\beta_j + q_j)$). Suppose

[8] We have checked that the derivations below are consistent with the requirement that our equilibria are stable.

that $t_{12}+t_{13} \geq \mu > t_{12}$, and that network 1 refuses to interconnect with network 2. In the absence of transit, the value of a message net of the delay cost is $v - \frac{k}{\mu - t_{12}}$, which is close to v (that is , $\theta \equiv 1 - \frac{k}{v(\mu - t_{12})}$ is close to 1) if k is small. In contrast, in the presence of transit, the equilibrium total traffic at the interface, t, is lower than $t_{12}+t_{13}$ and is such that interconnection with network 1 is worthless : $v - \frac{k}{\mu - t_{12}} = 0$. In other words, the gain from connectivity is completely dissipated by the customers' competition for sparse interface capacity. Hence, transit is equivalent (in utilities, although not in outcomes) to the absence of interconnection between networks 1 and 2 ($\theta_{12} = 0$).

4.1. No merger: Competition among equals

We first analyze the outcome of the pre-merger competition between the 4 symmetric backbones ($\beta_i = \beta/4$). Building on the analysis of the case of a duopoly, and letting θ_{ij} denote the quality of interconnection between backbones i and j, backbone i's profit function is

$$\begin{aligned} \pi_i &= \left[1 - q_i - \sum_{j \neq i} q_j + v\left(\frac{\beta}{4} + q_i + \sum_{j \neq i} \theta_{ij}\left(\frac{\beta}{4} + q_j\right)\right) - c\right] q_i \\ &= (1-v)\left(M_i - \sum_{j \neq i} K_{ij} q_j - q_i\right) q_i, \end{aligned}$$

where

$$\begin{aligned} M_i &= \frac{1 - c + \left(1 + \sum_{j \neq i} \theta_{ij}\right) v \frac{\beta}{4}}{1-v}, \\ K_{ij} &= \frac{1 - v\theta_{ij}}{1-v}. \end{aligned}$$

Hence, the best response function is given by

$$R_i(q_j, q_k, q_l) = \frac{M_i - \sum_{j \neq i} K_{ij} q_j}{2},$$

from which can be derived the equilibrium capacities and profits. Intuitively, since the backbones' installed bases all have the same size, the backbones will share the same incentive to maintain the quality of interconnection, and this can be checked formally. If, say, backbones 2, 3 and 4 maintain a perfect interconnection between themselves ($\theta_{23} = \theta_{24} = \theta_{34} = 1$), a given choice of $(\theta_{12}, \theta_{13}, \theta_{14})$ leads in equilibrium to a capacity and a profit for backbone 1 which both decrease if the quality of any interface (θ_{12}, θ_{13} or θ_{14}) is decreased. As a consequence, in the absence of interconnection costs, in equilibrium all backbones maintain the highest quality[7] and their profit is

$$\pi_i^* = (1-v)(q_i^*)^2 = \frac{(1-c+v\beta)^2}{25(1-v)}.$$

4.2. Post-merger accommodation strategy

A merger between backbones 1 and 4 creates a "dominant backbone", labelled backbone 1, with an installed base of size $\beta/2$, competing with two smaller backbones (backbones 2 and 3), each with an installed base of size $\beta/4$.

If the dominant backbone adopts the accommodation strategy ($\theta_{ij} = 1$), then the other two optimally follow suit and each backbone i maximizes

$$[1 - q_i - q_{-i} + v(\beta + q_i + q_{-i}) - c]\, q_i,$$

and thus, for $i = 1, 2, 3$:[8]

$$\begin{aligned} R_i(q_{-i}) &= \frac{1-c+v\beta-(1-v)\,q_{-i}}{2(1-v)}, \\ q_i^a &\equiv \frac{1-c+v\beta}{4(1-v)}, \\ \pi_i^a &\equiv (1-v)(q_i^a)^2, \end{aligned}$$

where the superscript a stands for "accommodation". Note that in this situation the dominant backbone loses the strategic benefit of its installed base.

[7] The analysis is more complex if we take into account the fact that a backbone could react to a degradation of a connection by negotiating a transit agreement with another backbone. One can show that the results presented here still hold.

[8] Note that the dominant backbone gets in that case less than in the pre-merger situation ($\pi_1^a < \pi_1^*$), as is standard in Cournot oligopoly models: We know since the work of S. W. Salant, S. Switzer and R. J. Reynolds (1983), "Losses from horizontal merger: The effects of an exogenous change in industry structure on Cournot-Nash equilibrium", *Quarterly Journal of Economics*, 48:185-199, that in a Cournot framework a merger may well increase the aggregate profits of the industry but lower the profits of merging firms. *The Cournot approach thus offers a rather pessimistic view regarding the profitability of a merger.* As can be checked below, in the confine of our model the merger can only be profitable when the dominant backbone can afterwards abuse of its position through degradation strategies. Other models, based on price competition rather than on quantity or capacity competition, exhibit higher profitabilities for the merging entities, since the merger then results in higher prices and profits for the merging firms as well as for the rest of the industry.

4.3. Global degradation strategy

If backbone 1 opts for the global degradation strategy ($\theta_{12} = \theta_{13} = 0, \theta_{23} = 1$), it maximizes

$$\left[1 - q_1 - q_{-1} + v\left(\frac{\beta}{2} + q_1\right) - c\right] q_1,$$

which yields

$$R_1(q_2 + q_3) = \frac{1 - c + v\frac{\beta}{2} - (q_2 + q_3)}{2(1-v)},$$

whereas backbone $i \in \{2,3\}$ maximizes

$$\left[1 - q_i - q_1 - q_j + v\left(\frac{\beta}{2} + q_i + q_j\right) - c\right] q_i,$$

yielding

$$R_i(q_1, q_j) = \frac{1 - c + v\frac{\beta}{2} - q_1 - (1-v)\, q_j}{2(1-v)}.$$

The equilibrium values are then for the dominant backbone:

$$\begin{aligned} q_1^g &= \frac{(1-3v)\left(1 - c + \frac{v\beta}{2}\right)}{2(2 - 6v + 3v^2)}, \\ \pi_1^g &= (1-v)\,(q_1^g)^2, \end{aligned}$$

with g for "global degradation", and for each of the smaller ones ($i = 2, 3$):

$$\begin{aligned} q_i^g &= \frac{(1-2v)\left(1 - c + \frac{v\beta}{2}\right)}{2(2 - 6v + 3v^2)}, \\ \pi_i^g &= (1-v)\,(q_i^g)^2. \end{aligned}$$

We now check that the dominant backbone never prefers to degrade the interconnection quality: $\pi_1^g < \pi_1^*$ is equivalent to

$$(1-c)(2-3v) + \beta(1-2v) > 0,$$

which is indeed always satisfied.

Remark. That the dominant backbone would not opt for global degradation is easy to prove in the case where the two small backbones can coordinate their capacity decision, so as to maximize their joint profits. In that situation, the final result would be the same as in a duopoly with equal installed bases, $\beta_1 = \beta_{23} = \beta/2$, with "$\theta = 1$" in the case of accommodation, yielding

$$\pi_1^{a,c} \equiv \frac{(1-c+v\beta)^2}{9(1-v)},$$

and "$\theta = 0$" in the case of global degradation, yielding

$$\pi_1^{g,c} \equiv \frac{1-v}{9}\left(\frac{1-c+v\beta/2}{1-2v/3}\right)^2,$$

which is always smaller than $\pi_1^{a,c}$: as we have already observed, "equals" prefer to maintain the best interconnection between them.

4.4. Divide and conquer: the strategy of targeted degradation

Consider now the strategy of targeted degradation, where backbone 1 does not interconnect with backbone 3 ($\theta_{13} = 0$) but maintains a high quality of interface with backbone 2 ($\theta_{12} = 1$) as long as this backbone does not offer transit to backbone 3. On the other hand, the capacity of the interface between backbones 1 and 2 is limited, so that this interface is fully degraded if backbone 2 accepted to serve as a conduit for traffic between 3 and 1.

Let us first consider backbone 2's transit decision. If it enters a transit agreement with backbone 3, the situation that prevails is the same as in the case of global degradation, which we have already examined. If instead backbone 2 does not offer transit to the other small backbone, then the (inverse) demands for the three backbones are respectively given by:

$$\begin{aligned}
p_1 &= 1 + v\left(\frac{3\beta}{4} + q_1 + q_2\right) - q_1 - q_2 - q_3, \\
p_2 &= 1 + v(\beta + q_1 + q_2 + q_3) - q_1 - q_2 - q_3, \\
p_3 &= 1 + v\left(\frac{\beta}{2} + q_2 + q_3\right) - q_1 - q_2 - q_3;
\end{aligned}$$

backbones 1 and 2 thus benefit from a better interconnection quality than backbone 3 does (as it is perfectly connected with the entire installed base, backbone 2 actually enjoys an even better quality than backbone 1 does). As a result, backbone 3's market share is much reduced, which may benefit the other two backbones. The equilibrium strategies, if interior ($q_i > 0$ for all i) satisfy:

$$q_1 = \frac{1 - c + v\frac{3\beta}{4} - (1-v)\,q_2 - q_3}{2(1-v)},$$

$$q_2 = \frac{1 - c + v\beta - (1-v)\,q_1 - (1-v)\,q_3}{2\,(1-v)},$$

$$q_3 = \frac{1 - c + v\frac{\beta}{2} - q_1 - (1-v)\,q_2}{2\,(1-v)}.$$

This enables us to compute the equilibrium quantities:

$$\begin{aligned}
q_1^t &\equiv \frac{(1-2v)\,(1-c) + 3\,(1-v)\,\frac{v\beta}{4}}{2\,(2-5v+2v^2)},\\
q_2^t &\equiv \frac{1-c+(7-3v)\,\frac{v\beta}{4}}{2\,(2-v)\,(1-v)},\\
q_3^t &\equiv \frac{(1-2v)\,(1-c) - \frac{v\beta}{4}\,(1+v)}{2\,(2-v)\,(1-2v)},
\end{aligned} \tag{4.1}$$

where t stands for "targeted degradation".

For conciseness, we will be mainly interested in the case in which backbone 3 is not able in equilibrium to attract any new customer (i.e. $q_3 = 0$). This will be the case when the q_3^t of equation (4.1) is negative, i.e. when

$$\frac{v\beta}{4} > \frac{(1-2v)\,(1-c)}{1+v}. \tag{4.2}$$

Then, the equilibrium capacity q_1 and q_2 satisfy:

$$q_1 = \frac{1 - c + v\frac{3\beta}{4} - (1-v)\,q_2}{2\,(1-v)},$$

$$q_2 = \frac{1 - c + v\beta - (1-v)\,q_1}{2\,(1-v)},$$

and the equilibrium is

$$\bar{q}_1^t = \frac{1}{6}\frac{2\,(1-c)+v\beta}{1-v},$$

$$\bar{q}_2^t = \frac{1}{12}\frac{4\,(1-c)+5v\beta}{1-v}.$$

and $\bar{\pi}_i^t = (1-v)\,(\bar{q}_i^t)^2$.

It can be checked that *the targeted degradation strategy always induces the small backbone 2 to refuse transit services to the other small backbone*[9]. *Backbone 1 prefers this targeted degradation strategy to accommodation (even if a higher quality of interconnection is costless) if*

$$\tilde{\pi}_1^t > \pi_1^a \Leftrightarrow v\beta < 1 - c.$$

This inequality is compatible with (4.2) if

$$4\frac{(1-2v)(1-c)}{1+v} < v\beta < 1 - c,$$

that is, if

$$v > \frac{1}{3}.$$

Remark (multihoming) Would customers want to multihome to backbones 1 and 3? (There is no point multihoming to backbone 2 and another backbone, because backbone 2 provides perfect connectivity). Using $p_3 = c$, the reader will check that in equilibrium it is possible to have $p_1 + p_3 > p_2$, and so to obtain perfect connectivity a customer is better off connecting to network 2 rather than to multihoming. Hence, as in the duopoly case, multihoming does not arise in this model (and usage-based pricing would not change this conclusion).

Remark. Targeted degradation may be optimal for the dominant network even if network 3 still attracts some new consumers after degradation. One can show that in this case, it is still optimal for backbone 2 not to provide transit to backbone 3.

Remark. Degradation would be even more appealing to the dominant backbone if

- the quality of interconnection were costly, and/or
- the dominant backbone could auction off between backbones 2 and 3 the "privilege" of not being targeted.

[9] Indeed:

$$\begin{aligned} \tilde{\pi}_2^t - \pi_2^a &= \frac{1}{12}\frac{4(1-c)+5v\beta}{1-v} - \frac{1-c+v\beta}{4(1-v)} \\ &= \frac{1}{12}\frac{1-c+2v\beta}{1-v} > 0. \end{aligned}$$

Appendix 2: Internet services, on-net substitution and the value of interconnection

22 April 1998

1 Introduction

The aim of this appendix is to study the incentives of a dominant Internet backbone to degrade quality, *at given market shares.* (Appendix 1 focuses on competition for market share, ignoring the substitution effect studied in this appendix). An intuitive argument indicates that, by limiting the quality of connection with the rest of the network, the dominant backbone induces its customers to communicate more intensely with each other, thus increasing traffic on its network and increasing its revenues. There is a seemingly straightforward counterargument, which seems to hold even for very large networks: by doing so the dominant network decreases the utility that its own customers derive from the services that it offers, and hence reduces its own revenues. Indeed, according to this counterargument, even though each of its customers has little communication with customers of other networks, the aggregate cost of degradation of the interconnection, aggregated over many customers, is large.

We show that this counterargument has limited validity for the Internet. We build a model with both users and suppliers of Internet services (one can think of dial up customers and Web sites, or of teenagers and game sites, and so on). Users have both a preferred supplier and a second choice supplier. We assume that there are two networks (it should be obvious that the same reasoning would hold with more). If the preferred supplier of a user is on the other network and if the quality of interconnection is bad enough, she prefers to visit her second choice supplier, as long as it is on the same network to which she is connected. Using this very natural model of the demand for services we show the following properties:

1. An increase in the quality of interconnection increases the revenue that the large network derives from users less than it increases the revenue of the smaller network. As a consequence, the large network has less incentives to invest in quality.

2. An increase in the quality of interconnection *decreases* the revenue that the large network derives from suppliers, whereas it increases the revenue that the small network derives from suppliers. On this ground, the *large network has a positive incentive to degrade quality.*

3. As a consequence, the large network always has less than socially optimal incentives to invest in quality, and if the weight of the revenue from suppliers of services is important enough, it can actually have incentives to degrade quality, even if this does not reduce costs.

2 A simple model with no substitution

We consider an Internet to which are connected n users of services and m suppliers. We will think of both n and m as being very large, with n a few orders of magnitude greater than m. The Internet is composed of two networks, network 1 and network 2. A proportion α_1 of the users and the same proportion α_1 of the suppliers are on network 1, while proportion $\alpha_2 = 1-\alpha_1$ of both users and suppliers are connected to network 2. The assumption that network 1 is the large network is formally stated as $\alpha_1 > 1/2 > \alpha_2$.

Remark. In order to simplify the notation, we assume that the same proportion of suppliers and visitors are in each network. As explained in the conclusion, this assumption can easily be dropped.

Assume first that each user has a single supplier to which she would like to connect. If the supplier is on-net she gets a utility of 1 (this is just a normalization), while if the supplier is off-net she gets a utility of $\theta \leq 1$. The parameter θ represents the quality of the interconnection. We assume that the preferences of users among suppliers are independent of their location, so that any user has *a priori* a probability $1/m$ of preferring any given supplier.

Then, the average utility of users of network 1 is

$$\alpha_1 \times 1 + \alpha_2 \times \theta = 1 - \alpha_2(1-\theta). \tag{1}$$

Indeed, a proportion α_1 of the users of network 1 have their preferred supplier on-net and get a utility of 1, and a proportion $\alpha_2 = 1-\alpha_1$ have their preferred supplier off-net and get a utility of θ. As consequence the total surplus of visitors of network 1 is

$$\begin{aligned} n\alpha_1(1-\alpha_2(1-\theta)) &= n(\alpha_1 - \alpha_1\alpha_2(1-\theta)) \\ &= n(\alpha_1^2 + \alpha_1\alpha_2\theta). \end{aligned}$$

Assume that the network succeeds in obtaining a revenue equal to λ times the total surplus that the visitors obtain from the services they obtain. Then, an increase $d\theta$ in quality yields for network 1 an increase in revenue $n\alpha_1\alpha_2\lambda d\theta$.

Similarly the average utility of users on network 2 is

$$1 - \alpha_1(1-\theta),$$

and the total surplus of users on network 2 is

$$\alpha_2^2 + \alpha_1\alpha_2\theta.$$

An increase in quality yields the same increase in revenue coming from users for network 2 as it does for network 1.

Of course, in this model, one needs also to consider the surplus obtained by the suppliers of services. For simplicity, we will assume that the suppliers are willing to pay a certain amount per "hit", i.e., per user that connects. (One would obtain the same results if we assumed that the surplus of the suppliers is dependant on the utility of the users who connect.)

The total number of hits for each supplier is n/m, and is clearly independent of its location. A change in the quality of interconnection does not affect the suppliers, and the thesis according to which the two networks have the same incentives to increase quality also holds on that side of the market.

Hence, in this very simple framework, it is correct that dominance does not provide incentives for degradation (assuming, as we do throughout this appendix, that market shares are rigid). We will see however that this conclusion is reversed as soon as there exists competition among suppliers.

3 Substitution and degradation

The aim of this section is to show that the conclusion of the previous section only holds because of the extremely simplified and unrealistic assumption that we have made about demand. If users are willing to trade-off content for quality of connection, as they obviously are, networks of different sizes have different incentives to improve the quality of interconnection.

In order to explore this effect, we keep the same assumptions about the distribution of users and suppliers, but we modify slightly the assumptions about the utility of the users. Now, each of them has two acceptable suppliers: connection to the preferred supplier yields a utility of 1, while connection to the second best supplier yields a utility of $v \in [0,1]$ (different users will have different values of v). For simplicity, we assume that v is uniformly distributed on $[0,1]$, and furthermore that it is independent of the location of the user.

3.1 Users

3.1.1 The utility of users

There are three relevant cases to study the behavior of a user:

- If her favorite supplier is on-net, she connects to it and obtains a utility of 1.
- If both her favorite and second-best suppliers are off-net, she connects to her favorite supplier and obtains a utility of θ.
- If her favorite supplier is off-net and her second best supplier is on-net, she compares the utility that she derives from the favorite supplier, θ, and the utility that she derives from her second best supplier, v:
    - if[1] $\theta \geq v$, she connects to her first best supplier;
    - if $\theta < v$, she connects to her second best supplier.

[1] We assume that users connect to their favorite supplier when they are indifferent, but this has absolutely no impact on the results.

A user on network 1, the large network, for whom $v \leq \theta$ (she prefers to connect to her preferred supplier off-net than to her second choice supplier on-net) has an expected utility of

$$\alpha_1 \times 1 + \alpha_2 \times \theta = 1 - \alpha_2(1-\theta).$$

This is the same expression as in (1): the user chooses to connect to her favorite supplier, and faces a loss of utility of $1-\theta$ when this supplier if off-net.

A user on network 1 for whom $v > \theta$ (she prefers to connect to her second choice supplier on-net than to her preferred supplier off-net), has an expected utility of (approximately,[2] for m large):

$$\alpha_1 + (1-\alpha_1)(\alpha_1 v + \alpha_2\theta).$$

Indeed, with probability α_1 her favorite supplier is on-net and she gets a utility of 1. Conditional to the fact that the favorite supplier is off-net, there is a probability α_1 that the second best supplier is on-net , and the utility of the user is v in these cases. With probability α_2 both of her acceptable suppliers are off-net , and her utility is θ.

We now compute the average utility of the users of network 1. (Remember that the parameter v is uniformly distributed between 0 and 1, and is independent of the location of the acceptable suppliers of the user.) This

[2]Without this approximation her utility is

$$\alpha_1 + (1-\alpha_1)\left(\frac{\alpha_1 m}{m-1}v + \frac{\alpha_2 m - 1}{m-1}\theta\right).$$

Indeed, with probability α_1 her favorite supplier is on-net and she gets a utility of 1. If the favorite supplier is off-net, there is a probability

$$\frac{\alpha_1 m}{m-1}$$

that the second best supplier is on-net (there are $m-1$ potential second best suppliers, and $\alpha_1 m$ of them are on network 1), and the utility of the user is v in these cases. With probability

$$\frac{\alpha_2 m - 1}{m-1} = 1 - \frac{\alpha_1 m}{m-1}$$

both of her acceptable suppliers are off-net (of the $m-1$ potential second best suppliers, $\alpha_2 m - 1$ are on network 2), and her utility is θ.

The expression in the text is obtained as the limit when m goes to infinity.

average utility is

$$\theta(1-\alpha_2(1-\theta)) + \int_\theta^1 (\alpha_1 + (1-\alpha_1)(\alpha_1 v + \alpha_2\theta))\, dv$$
$$= \alpha_1 + \theta\alpha_2 + \frac{(1-\theta)^2}{2}\alpha_1\alpha_2$$
$$= \frac{1}{2}\alpha_1\alpha_2\theta^2 + \alpha_2^2\theta + \alpha_1\frac{3-\alpha_1}{2}.$$

Nothing that we have done up to this point depends on the fact that network 1 is the largest network, so that the average utility of a user in network 2 is

$$\alpha_2 + \theta\alpha_1 + \frac{(1-\theta)^2}{2}\alpha_1\alpha_2$$
$$= \frac{1}{2}\alpha_1\alpha_2\theta^2 + \alpha_1^2\theta + \alpha_2\frac{3-\alpha_2}{2}.$$

The total utility of users of network 1 is

$$V_1(\theta) = \alpha_1 n\left(\frac{1}{2}\alpha_1\alpha_2\theta^2 + \alpha_2^2\theta + \alpha_1\frac{3-\alpha_1}{2}\right),$$

and the total utility of users of network 2 is

$$V_2(\theta) = \alpha_2 n\left(\frac{1}{2}\alpha_1\alpha_2\theta^2 + \alpha_1^2\theta + \alpha_2\frac{3-\alpha_2}{2}\right).$$

3.1.2 Quality choice by the networks

Given the demand patterns that we have just studied, we now turn to the quality choice by the networks. In this section, we assume that their revenue is proportional to the utility of the users, and reason as if the revenue provided by the suppliers were nil. As will be seen in the next section, lifting this restrictive hypothesis will only reinforce our conclusions.

Relative preferences for quality of interconnection Notice first that both V_1 and V_2 are increasing in θ so that both networks would prefer high quality, if it were free. However, if the cost is increasing in θ, network 1 will prefer a lower quality. To see this, note that

$$V_1'(\theta) = \alpha_1 n\left(\alpha_1\alpha_2\theta + \alpha_2^2\right)$$

and

$$V_2'(\theta) = \alpha_2 n \left(\alpha_1\alpha_2\theta + \alpha_1^2\right).$$

We obtain

$$\begin{aligned} V_1'(\theta) - V_2'(\theta) &= n(\alpha_1 - \alpha_2)(\alpha_1\alpha_2\theta - \alpha_1\alpha_2) \\ &= n(\alpha_1 - \alpha_2)\alpha_1\alpha_2(\theta - 1) \\ &< 0. \end{aligned} \tag{2}$$

Hence, for any θ network 1 has less incentives to increase quality than does network 2. Because the functions V_i are convex, standard comparative statics results do not apply and it requires a little bit more work to show that the preferred level of quality of network 1 is lower than the preferred level of quality of network 2.

Proposition 1 *Assume that the cost function $c(\theta)$ is differentiable. Then the preferred quality θ_1^* of network 1 is smaller than the preferred quality θ_2^* of network 2.*

Proof. By definition we must have (remember that a network receives as revenue a proportion λ of the utility of the users):

$$\begin{aligned} \lambda V_1(\theta_1^*) - c(\theta_1^*) &\geq \lambda V_1(\theta_2^*) - c(\theta_2^*), \\ \lambda V_2(\theta_2^*) - c(\theta_2^*) &\geq \lambda V_2(\theta_1^*) - c(\theta_1^*). \end{aligned}$$

Adding up, this implies

$$V_1(\theta_1^*) - V_1(\theta_2^*) \geq V_2(\theta_1^*) - V_2(\theta_2^*),$$

which is equivalent to

$$\int_{\theta_2^*}^{\theta_1^*} V_1'(\theta)\, d\theta \geq \int_{\theta_2^*}^{\theta_1^*} V_2'(\theta)\, d\theta.$$

By (2), this implies $\theta_2^* > \theta_1^*$.

Note that if at least one of θ_1^* and θ_2^* is strictly between 0 and 1, we have $\theta_1^* < \theta_2^*$, as the first order conditions cannot be met for both networks at the same quality. ■

3.1.3 Equilibrium quality of interconnection

Proposition 1 does not establish that an increase in the size of network 1 will lead to a decrease in quality. Indeed, it leaves open the possibility that an increase in α_1 would increase both θ_1^* and θ_2^*, with $\theta_1^* < \theta_2^*$. Furthermore, we have not explicitly described the way in which quality is be determined when the two networks have different preferred levels.

In order to prove that an increase in α_1 (and the concurrent decrease in α_2) decreases the quality of interconnection, we develop two possible scenarios for its determination.

Veto power on quality We first assume that the quality is determined by the network that desires the smallest quality (this is a natural assumption for a market in which each of the of networks can unilaterally refuse to invest in the quality of the interface). In this sense, either of the two networks has a veto power on any increase in the quality of interconnection.

Under these circumstances, it is network 1 which will choose the quality θ of interconnection. To show that an increase in α_1 and the corresponding decrease in α_2 induce a decrease in equilibrium quality, it is sufficient to show that the preferred quality of network 1 decreases when α_1 increases. To see this notice that

$$\begin{aligned}\frac{1}{n}\frac{\partial V_1}{\partial\theta\partial\alpha_1} &= \frac{1}{n}\frac{\partial\left[\alpha_1(1-\alpha_1)(\alpha_1\theta+1-\alpha_1)\right]}{\partial\alpha_1} \\ &= 1-4\alpha_1+3\alpha_1^2+\theta\alpha_1(2-3\alpha_1).\end{aligned}$$

When $\theta = 1$ this quantity is equal to $1-2\alpha_1 < 0$. When $\theta = 0$, it is equal to $1-4\alpha_1+3\alpha_1^2$, which is strictly negative (equal to $-1/4$) for $\alpha_1 = 1/2$ and equal to 0 for $\alpha_1 = 1$, and hence, by strict concavity, is also strictly negative for any $\alpha_1 \in [1/2, 1)$. Therefore $\partial V_1/\partial\theta\partial\alpha_1$ is negative for $\theta = 0$ and $\theta = 1$, and hence, because it is linear in θ, it is also negative for all $\theta \in (0, 1)$.

A reasoning exactly similar to that of the proof of proposition 1 is then sufficient to prove that when α_1 increases, the quality preferred by network 1 decreases, which proves the following proposition.

Proposition 2 *If the equilibrium quality is equal to the smallest preferred quality of the networks, the equilibrium quality decreases as α_1 increases.*

Networks agree on quality A natural objection to proposition 2 is that if the networks disagree on the quality, they will bargain over the quality of the interconnection, the smaller one, if necessary, subsidizing the larger one. We are unsure about the importance of this argument in the case of the Internet, where quality is partly "non-contractible". However, even if it can be contracted upon, an increase in α_1 leads to a decrease in the quality of interconnection.

The players choose the quality that maximizes the sum of their profits. Hence, they agree on the quality θ of interconnection that maximizes

$$\lambda V_1(\theta) + \lambda V_2(\theta) - 2c(\theta). \tag{3}$$

We have

$$\begin{aligned} V_1(\theta) + V_2(\theta) &= \alpha_1 n \left(\frac{1}{2}\alpha_1\alpha_2\theta^2 + \alpha_2^2\theta + \alpha_1\frac{3-\alpha_1}{2}\right) \\ &\quad + \alpha_2 n \left(\frac{1}{2}\alpha_1\alpha_2\theta^2 + \alpha_1^2\theta + \alpha_2\frac{3-\alpha_2}{2}\right) \\ &= \alpha_1\alpha_2 n \left(\frac{\theta^2}{2} + \theta\right) + \alpha_1^2 n\frac{3-\alpha_1}{2} + \alpha_2^2 n\frac{3-\alpha_2}{2}. \end{aligned}$$

Therefore

$$\frac{d(V_1(\theta) + V_2(\theta))}{d\theta} = \alpha_1\alpha_2 n(\theta + 1).$$

Because this quantity decreases when α_1 increases (and α_2 decreases), the θ that maximizes (3) decreases when α_1 increases (again appealing to the same reasoning as in the proof of proposition 1).

We have therefore proved the following proposition:

Proposition 3 *If the equilibrium quality is determined by bargaining between the networks, the equilibrium quality decreases as α_1 increases.*

3.2 Suppliers

We now turn to the study of the incentives to increase the quality of interconnection in order to increase the value of the networks to suppliers of services. For this section, we assume that the revenue of a network is proportional to the number of "hits" on its suppliers, that is to the number of users that connect to them.

Consider a supplier of services, let us call it A, on network 2. There will be

$$\frac{n}{m}\alpha_2$$

users who will connect to A and get a utility of 1 ($n\alpha_2$ users are on network 2 and $1/m$ of those have A as their first choice).

Some users will connect to A because their favorite supplier is on network 1, but they prefer to connect on-net to their second choice. There are

$$n\alpha_2 \times \alpha_1 \times \frac{1}{m-1} = \frac{n\alpha_1\alpha_2}{m-1}$$

users connected to network 2 whose first choice is connected to network 1 and who have A as their second choice. ($n\alpha_2$ users on network 2, a proportion α_1 of those have their first choice supplier on network 1 and $1/(m-1)$ of those have A as their second choice). Among those users, only those for which $v \geq \theta$ will connect to A. Therefore the total number of on-net users who connect to A as their second choice is

$$\frac{n\alpha_1\alpha_2}{m-1} \times (1-\theta).$$

There are

$$\frac{n\alpha_1}{m} \times \frac{m\alpha_2 - 1}{m-1} = \frac{n\alpha_1(m\alpha_2 - 1)}{m\,(m-1)}$$

users who connect to A even though they are clients of network 1, because A is their first choice and their second choice is also on network 2 ($n\alpha_1/m$ users who are on network 1 and have A as their first choice, and a proportion[3] $(m\alpha_2 - 1)/(m-1)$ of those have their second choice on network 2). These users have a utility of θ.

Finally, some users are connected to network 1, and have their second choice on that network, but still prefer to use A because their v is small. There are

$$\frac{n\alpha_1}{m} \times \frac{m\alpha_1}{m-1} \times \theta = \frac{n}{m-1}\alpha_1^2\theta$$

of those ($n\alpha_1/m$ users are on network 1 and have A as their first choice, a proportion $m\alpha_1/(m-1)$ of those who have their second choice on network 1, and only those users for which $v < \theta$ choose to connect off-net).

[3] There are $m-1$ suppliers who are candidates to be the second choice supplier, and $m\alpha_2 - 1$ of them are on network 2.

The total number of hits for a supplier on network 2 will therefore be

$$\begin{aligned}
&\frac{n}{m}\alpha_2 + \frac{n\alpha_1\alpha_2}{m-1}(1-\theta) + \frac{n\alpha_1(m\alpha_2 - 1)}{m(m-1)} + \frac{n}{m-1}\alpha_1^2\theta \\
&\quad = \frac{n}{m(m-1)}((m-1)\alpha_2 + m\alpha_1\alpha_2(1-\theta) + \alpha_1(m\alpha_2 - 1) + m\alpha_1^2\theta) \\
&\quad = \frac{n}{m(m-1)}(2m\alpha_1\alpha_2 + m\alpha_2 - 1 + \theta m\alpha_1(\alpha_1 - \alpha_2)) \qquad (4)
\end{aligned}$$

connections to A.

Similarly, the number of hits for a supplier on network 1 will be (nothing in the reasoning leading to (4) depends on the fact that network 2 is the smallest network)

$$\frac{n}{m(m-1)}(2m\alpha_1\alpha_2 + m\alpha_1 - 1 + m\alpha_2\theta(\alpha_2 - \alpha_1)).$$

We have[4]

$$\alpha_1 + \alpha_2\theta(\alpha_2 - \alpha_1) \geq \alpha_2 + \alpha_1\theta(\alpha_1 - \alpha_2),$$

and therefore we have proved the following proposition:

Proposition 4 *The number of hits for a supplier on network 1 is greater (strictly greater if $\theta < 1$) than the number of hits for a supplier on network 2.*

The total number of hits on network 2 will be

$$\begin{aligned}
H_2(\theta) &= \frac{n}{m(m-1)}(2m\alpha_1\alpha_2 + m\alpha_2 - 1 + \theta m\alpha_1(\alpha_1 - \alpha_2)m\alpha_2) \\
&= \frac{n\alpha_2}{m-1}(2m\alpha_1\alpha_2 + m\alpha_2 - 1 + \theta m\alpha_1(\alpha_1 - \alpha_2)),
\end{aligned}$$

and similarly the total number of hits on network 1 is[5]

$$H_1(\theta) = \frac{n\alpha_1}{m-1}(2m\alpha_1\alpha_2 + m\alpha_1 - 1 + \theta m\alpha_2(\alpha_2 - \alpha_1)).$$

Proposition 5 *An increase in the quality of interconnection decreases the total number of hits on network 1 and increases the total number of hits on network 2.*

[4] It suffices to prove that the inequality holds for $\theta = 0$ and (weakly) for $\theta = 1$, which is immediate.

[5] It is easy to check by addition of those two formulas that the total number of hits is indeed n, as it should be.

If the income of the networks is proportional to the number of hits, network 1 will prefer a lower quality. In fact, the θ that maximizes the revenue that network 1 derives from suppliers is 0, while the θ that maximizes the revenue that network 2 derives from suppliers is 1! (The result that network 1 prefers $\theta = 0$ is perhaps extreme, and is related to the fact that users are never discouraged and, as a consequence, the total number of hits is independent of the quality of the interconnection, although the users' utility is not. Nonetheless, it illustrates well the general thrust of the argument.)

We can interpret $H_1'(\theta)$ as the strength of the incentives of network 1 to degrade quality. It depends on the size of the θ coefficient in $H_1(\theta)$,

$$\alpha_1\alpha_2(\alpha_2 - \alpha_1) = \alpha_1(1 - \alpha_1)(1 - 2\alpha_1),$$

which is increasing for $\alpha_1 \leq 1/2 + \sqrt{3}/6 \simeq 0.79$. For a large range of values of α_1, the larger network 1, the greater its incentives to degrade the quality of interconnection!

3.3 Conclusion

We have shown that the use of a realistic model of the demand for Internet services leads to fundamentally different conclusions than those obtained through the simplest naive model of demand. A large network has lower incentives to upgrade the quality of interconnection with another network.

We have stopped short of a complete equilibrium analysis in which the incomes obtained both from suppliers and users would be aggregated, because the results would be straightforward given the analysis already done (whatever the relative weights given to the utility of users and the number of hits on the providers, network 1 would have incentives to degrade quality).

Furthermore, we have not embedded the model in a full equilibrium model, but the introduction of the elements explored in this appendix would only serve to reinforce the conclusion of our other appendix "A model of strategic Internet backbone interconnection": dominant backbones do pose a threat to connectivity!

Appendix 6

WorldCom/MCI's Domination of the Internet Backbone Market